SPACE- AND TIME- ADAPTIVE GRIDDING USING MRTD TECHNIQUE

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Abstract- The MRTD scheme is applied to the analysis of waveguide problems. Specifically, the field pattern and the S-parameters of a dielectric-loaded parallel-plate waveguide are calculated. The use of wavelets enables the implementation of a space- and time-adaptive gridding technique. The results are compared to those obtained by use of the conventional FDTD scheme to indicate considerable savings in memory and computational time.

I Introduction

Recently a new technique has been successfully applied [1-4] to a variety of microwave problems and has demonstrated unparalleled properties. This technique is derived by the use of multiresolution analysis for the discretization of the time-domain Maxwell's equations. The multiresolution time domain technique (MRTD) based on Battle Lemarie functions has been applied to linear as well as nonlinear propagation problems. The PML absorbing boundary condition has been generalized in order to analyze open planar structures. MRTD has demonstrated savings in time and memory of two orders of magnitude. In addition, the most important advantage of this new technique is its capability to provide space and time adaptive gridding without the problems that the conventional FDTD is encountering. This is due to the use of two separate sets of basis functions, the scaling and wavelets and the capability to threshold the field coefficients due to the excellent conditioning of the formulated mathematical problem.

In this paper, a space/time adaptive gridding algorithm based on the MRTD scheme is proposed and applied to the waveguide problems. As an example, the propagation of a Gabor pulse in a partially-filled parallel-plate waveguide is simulated and the S-parameters are evaluated. Wavelets are placed only at locations where the EM fields have significant values, creating a space- and time- adaptive dense mesh in regions of strong field variations, while maintaining a much coarser mesh elsewhere.

II The 2D-MRTD scheme

For simplicity the 2D-MRTD scheme for the TM_z modes will be used herein. To derive the 2D-MRTD scheme, the field components are represented by a series of cubic spline Battle-Lemarie [5] scaling and wavelet functions to the longitudinal direction in space and pulse functions in time. After inserting the field expansions in Maxwell's equations, we sample them using pulse functions in time and scaling/wavelet functions in space domain.

As an example, sampling $\partial D_x/\partial t, = -\partial H_y/\partial z$ in space and time, the following difference equation is obtained

$$\frac{1}{\Delta t}({}_{k+1}D^{\phi x}_{l+1/2,m} - {}_{k}D^{\phi x}_{l+1/2,m}) =$$

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$$-\frac{1}{\Delta y}\Big(\sum_{i=m-m_2}^{m+m_1} a(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2}$$

$$+\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2}\Big) \qquad , \qquad (1)$$

$$\frac{1}{\Delta t}({}_{k+1}D^{\psi x}_{l+1/2,m} - {}_{k}D^{\psi x}_{l+1/2,m}) =$$

$$-\frac{1}{\Delta y}\Big(\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2}$$

$$+\sum_{i=m-m_6}^{m+m_5} c(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2}\Big) \qquad , \qquad (2)$$

where ${}_kD^{\xi x}_{l,m}$ and ${}_kH^{\xi y}_{l,m}$ with ξ=ϕ (scaling),ψ (wavelets) are the coefficients for the electric and magnetic field expansions. The indices l, m and k are the discrete space and time indices, which are related to the space and time coordinates via $x = l\Delta x$,$z = m\Delta z$ and $t = k\Delta t$, where Δx,Δz are the space discretization intervals in x- and z-direction and Δt is the time discretization interval. The coefficients $a(i), b(i), c(i)$ are derived and given in [2]. For an accuracy of 0.1% the values $m_1 = m_5 = 8$,$m_2 = m_3 = m_4 = m_6 = 9$ have been used.

For open structures, the perfectly matched layer (PML) technique can be applied by assuming that the conductivity is given in terms of scaling and wavelet functions instead of pulse functions with respect to space [4]. The spatial distribution of the conductivity for the absorbing layers is modelled by assuming that the amplitudes of the scaling functions have a parabolic distribution. The MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region. Usually, 8-16 cells of PML medium with σ^E_{max}=0.4S/m provide reflection coefficients smaller than -90 dB.

In order to use a pulse excitation at $z = m\Delta z$ with respect to space and to obtain an excitation identical to an FDTD excitation, we decompose the pulse in terms of scaling and wavelet functions

$${}_kE^{pulse}_m \approx E_F(0, k\Delta t)$$

$$\Big(\sum_{i=-4}^{+4} c_\phi(i)\phi_{m+i} + \sum_{i=-4}^{+4} c_\psi(i)\psi_{m+i}\Big) \qquad (3)$$

where the coefficients $c_\phi(i), c_\psi(i)$ are given in Table 1 for $i \geq 0$. For $i < 0$ it is $c_\phi(-i) = c_\phi(i)$ and $c_\psi(i) = c_\psi(1-i)$. $E_F(0, k\Delta t)$ is the time dependence of the excitation. For $|i| \leq 4$, the above excitation components are superimposed to the field values obtained by the MRTD algorithm. For example, the total $E^\phi_{k,m+i}$ will be given by

$$E^\phi_{k,m+i}\Big|_{total} = E_F(0, k\Delta t)\; c_\phi(i) + E^\phi_{k,m+i}$$

Due to the nature of the Battle-Lemarie expansion functions, the total field is a summation of the contributions from the non-localized scaling and wavelet functions. For example, the total electric field $E_x(x_o, z_o, t_o)$ with $(k - 1/2)\,\Delta t < t_o < (k + 1/2)\,\Delta t$ is calculated in the same way with [2, 3] by

$$E_x(x_o, z_o, t_o) = \sum_{l',m'=-l_1}^{l_1} {}_kE^{\phi x}_{l'+1/2,m'}\, \phi_{l'+1/2}(x_o)\, \phi_{m'}(z_o)$$

$$+\sum_i \sum_{l',m'=-l_{2,i}}^{l_{2,i}} {}_kE^{\psi_i x}_{l'+1/2,m'}\, \phi_{l'+1/2}(x_o)\, \psi_{i,m'}(z_o)$$

where $\phi_m(x) = \phi(\frac{x}{\Delta x} - m)$ and $\psi_{i,m}(x) = \psi_i(\frac{x}{\Delta x} - m)$ represent the Battle-Lemarie scaling and i-resolution wavelet function respectively. For an accuracy of 0.1% the values $l_1 = l_{2,i} = 4$ have been used.

There are many different ways to take advantage of the capability of the MRTD technique to provide space and time adaptive gridding. In DSP, thresholding of the wavelet coefficients over a specific time- and space- window (5-10 points) contribute significant memory economy, but increase the implementation complexity and the execution time. The simplest way is to threshold the wavelet components to a fraction (usually $\leq$ 0.1%) of the scaling function at the same cell for each time-step. All components below this threshold are eliminated from the subsequent calculations. This is the simplest thresholding algorithm. It doesn't add any significant overhead in execution time, but it offers only a moderate (pessimistic) economy in memory (factor close to 2). Also, this algorithm allows for the dynamic memory allocation in its programming implementation.

III Applications of 2D-MRTD

The 2D-MRTD scheme is applied to the analysis of the partially-loaded parallel-plate waveguide of (Fig.1) for the frequency range 0-30GHz. For the analysis based on Yee's FDTD scheme, a 16×800 mesh is used resulting in a total number of 14400 grid points. When the structure is analyzed with the 2D-MRTD scheme, a mesh 2×100 (200 grid points) is chosen ($dx = 0.24\lambda_o$, $dz = 0.4\lambda_o$ for $f = 30GHz$). This size is based on the number of the scaling functions, since the wavelets are used only when and where necessary. The time discretization interval is selected to be identical for both schemes and equal to the 1/10 of the 2D-MRTD maximum Δt. For the analysis we use 8,000 time-steps. The waveguide is excited with a Gabor function 0-30GHz along a vertical line for the FDTD simulation and for a rectangular region for the MRTD simulations. In all cases, the front and back open planes are terminated with a PML region of 16 cells and σ^E_{max}=0.4S/m. The longitudinal distance between the excitation and the dielectric interface is chosen such that no reflections would appear before the Gabor function is complete.

The capability of the MRTD technique to provide space and time adaptive gridding is verified by thresholding the wavelet components to the 0.1% of the value of the scaling function at the same cell for each time-step. It has been observed that the accuracy by using only a small number of wavelets is equal to what would be achieved if wavelets were used everywhere. Though this number is varying in time, its maximum value is 22 out of a total of 100 to the z-direction (economy in memory by a factor of 28-30). In addition, execution time is reduced by a factor 4-5. For larger thresholds, the ringing effect due to the elimination of the wavelets deteriorates the performance of the algorithm. For example, using a threshold of 1% (6 out of a 100 wavelets to the z-direction) increases the error by a factor of 2.5.

The normal electric field is probed at a distance 10 cells away from the source and is plotted in (Fig.2) in time-domain. Comparable accuracy can be observed for the FDTD and the MRTD meshes. In addition, the reflection coefficient S_{11} is calculated by separating the incident and the reflected part of the probed field and taking the Fourier transform of their ratio (Fig.3). The results for 5 GHz (TEM propagation) are validated by comparison to the theoretical value obtained applying ideal transmission line theory [6] and are plotted at Table 2. The time- and space-adaptive character of the gridding is exploited in (Figs.4,5) which show that the wavelets follow the propagating pulses before and after the incidence to the dielectric interfaces and have negligible values elsewhere. The location and the number of the wavelet coefficients with significant values are different for each time-step, something that creates a dense mesh in regions of strong field variations, while maintaining a much coarser mesh for the other cells.

IV Conclusion

A space- and time- adaptive gridding algorithm based on a multiresolution time-domain scheme in two dimensions has been proposed and has been applied to the numerical analysis of a waveguide problem. The field pattern and the reflection coefficient have been calculated and verified by comparison to reference data. In comparison to Yee's conventional FDTD scheme, the proposed scheme offers memory savings by a factor of 5-6 per dimension maintaining a similar accuracy. The above algorithm can be effectively extended to three-dimension problems.

V Acknowledgments

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Table 1: Excitation Decomposition Coeffs

i	0	1	2	3	4
$c_\phi(i)$	0.915	0.038	0.010	−0.009	0.005
$c_\psi(i)$	−0.103	−0.103	0.121	−0.030	0.015

Table 2: S_{11} calculated by 2D-MRTD

	S_{11} (Ω)	Relative error
Analyt. Value [6]	0.4298	0.0%
16x800 FDTD	0.4283	-0.3%
2x100 MRTD	0.4360	+1.4%

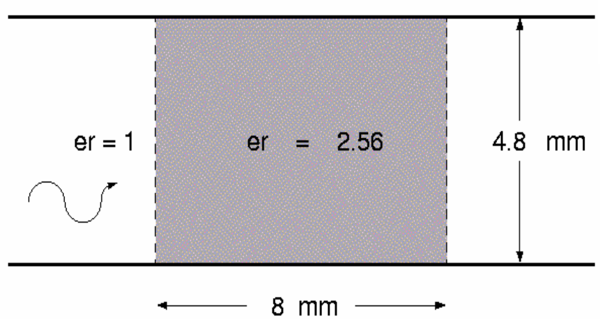

Figure 1: Dielectric-loaded Waveguide.

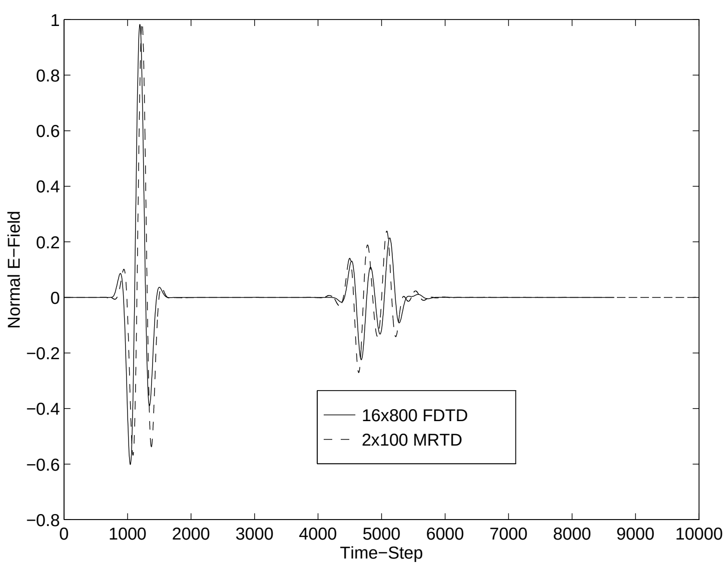

Figure 2: Normal E-field (Time-Domain).

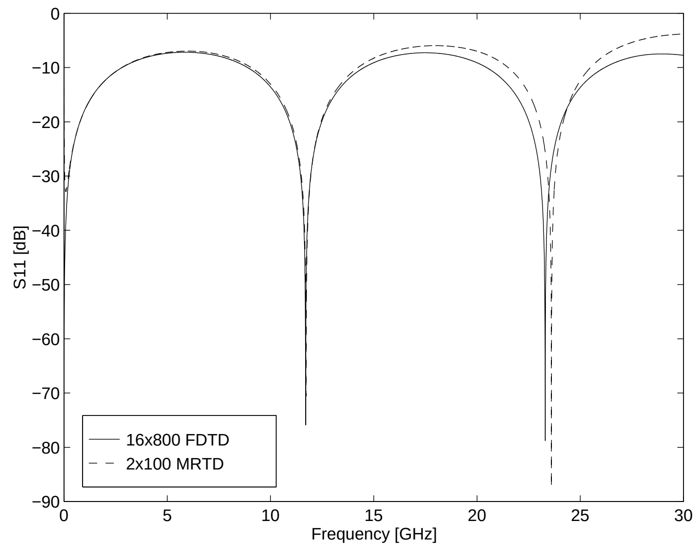

Figure 3: S_{11} values (Frequency-Domain).

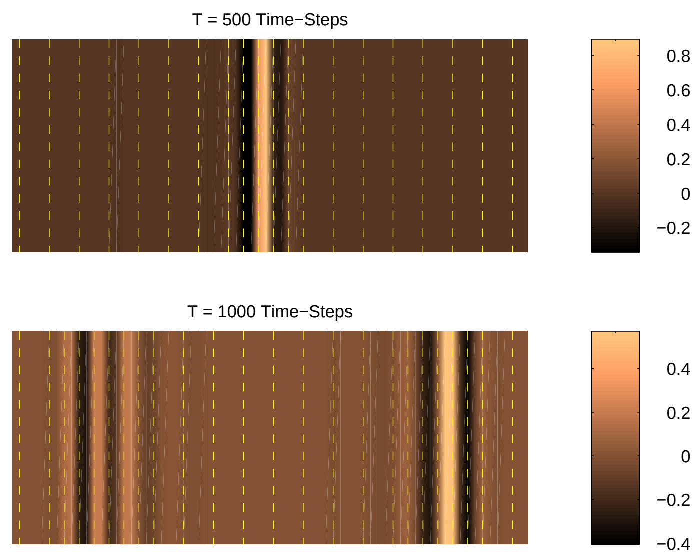

Figure 4: Adaptive Grid Demonstration.

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